VOLUME 77 SEPARATE No. 106

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

DECEMBER, 1951

ANALYSIS OF GROUND-WATER LOWERING ADJACENT TO OPEN WATER

By Stuart B. Avery

SOIL MECHANICS AND FOUNDATIONS DIVISION

Copyright 1951 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

Y620.6

GUIDEPOST FOR TECHNICAL READERS

"Proceedings-Separates" of value or significance to readers in various fields are here listed, for convenience, in terms of the Society's Technical Divisions. Where there seems to be an overlapping of interest between Divisions, the same Separate number may appear under more than one item.

Technical Division	*Proceedings-Separate Number*
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43)
City Planning	58, 60, 62, 64, 93, 94, 99, 101, 104, 105 (Discussion: D-16, D-23, D-43)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40)
Engineering Economics	46, 47, 62, 64, 65, 68, 69, 95, 100, 104 (Discussion: D-2, D-19, D-27, D-30, D-36)
Engineering Mechanics	41, 49, 51, 54, 56, 59, 61, 88, 89, 96 (Discussion: D-5, D-XXIII, D-XXV, D-18, D-24, D-33, D-34 D-49, D-54, D-61)
Highway	43, 44, 48, 58, 70, 100, 105, 108 (Discussion: D-XXVIII, D-7, D-13, D-16, D-23)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111 (Discussion: D-XXVII, D-9, D-11, D-19, D-28, D-29)
Irrigation	46, 47, 48, 55, 56, 57, 67, 70, 71, 87, 88, 90, 91, 96, 97, 98, 99, 102, 106, 109, 110, 111 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110 (Discussion: D-XXIII, D-2, D-3, D-7, D-11, D-17, D-19, D-25-K, D-30, D-38, D-40, D-44)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111 (Discussion: D-10, D-29, D-37)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109 (Discussion: D-3, D-5, D-8, D-13, D-16, D-17, D-21, D-23, D-24, D-25-K, D-32, D-33, D-34, D-37, D-39, D-42, D-49, D-54, D-61)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107 (Discussion: D-XXVII, D-8, D-9, D-19, D-27, D-28)

A constant effort is made to supply technical material to Society members, over the entire range of possible interest. Insofar as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement. Those who are planning papers for submission to "Proceedings-Separates" will expedite Division and Committee action measurably by first studying the ASCE "Guide for Development of Proceedings-Separates" as to style, content, and format. For a copy of this Manual, address the Manager, Technical Publications, ASCE, 33 W. 39th Street, New York 18, N. Y.

The Society is not responsible for any statement made or opinion expressed in its publications

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Proceedings—Vol. 77 December, 1951 Separate No. 106

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ANALYSIS OF GROUND-WATER LOWERING ADJACENT TO OPEN WATER

By Stuart B. Avery, Jr.[1]

Synopsis

In the construction of foundations for structures adjacent to open bodies of water, it is frequently necessary to lower the ground water by a system of wells. In such operations, it is highly desirable to know in advance the pumping capacity required to maintain the drawdown at particular elevations during normal and high water stages. This information can usually be obtained with the aid of pumping tests and mathematical analyses.

The purpose of this paper is to present formulas and graphs for the analysis of ground-water lowering and to demonstrate the use of such formulas and graphs. These mathematical relationships were developed for the purpose of determining pumping capacities required for dewatering a powerhouse site adjacent to a river.

Two sets of formulas have been derived; one for determining the relation between required pumping capacity and the drawdown at the center of a well installation; the other for determining the relation between the drawdown at the center of the well installation and the drawdown at any other point. With these formulas it is possible to determine:

(1) The rate of pumping required to maintain any desired drawdown with various arrangements of wells; and

(2) The change in rate of pumping required for any change in drawdown, arrangement of wells, or water stage.

In order to simplify the analysis, well installations have been considered to be circular, rather than having the usual square or rectangular arrangement.

Note.—Written comments are invited for publication; the last discussion should be submitted by June 1, 1952.

[1] Lecturer on Foundation Engineering, Harvard Univ.; Cons. Eng., Cambridge, Mass.

Derivation of Formulas

Basic Formula and Assumptions.—Phillip Forchheimer[2] has derived (by the method of images) a general equation for the drawdown due to pumping from a series of wells adjacent to an open body of water with a straight shore line. This condition is shown in Fig. 1. The Forchheimer equation is:

$$z^2 = H^2 - \frac{1}{\pi k}\left(q_1 \log_e \frac{S_1}{R_1} + q_2 \log_e \frac{S_2}{R_2} + \cdots + q_n \log_e \frac{S_n}{R_n}\right) \ldots\ldots (1)$$

in which z is the height of water surface above impervious boundary; H is the height of water surface above impervious boundary prior to pumping; k is the effective coefficient of permeability; $q_1, q_2, \ldots q_n$ is the rate of pumping

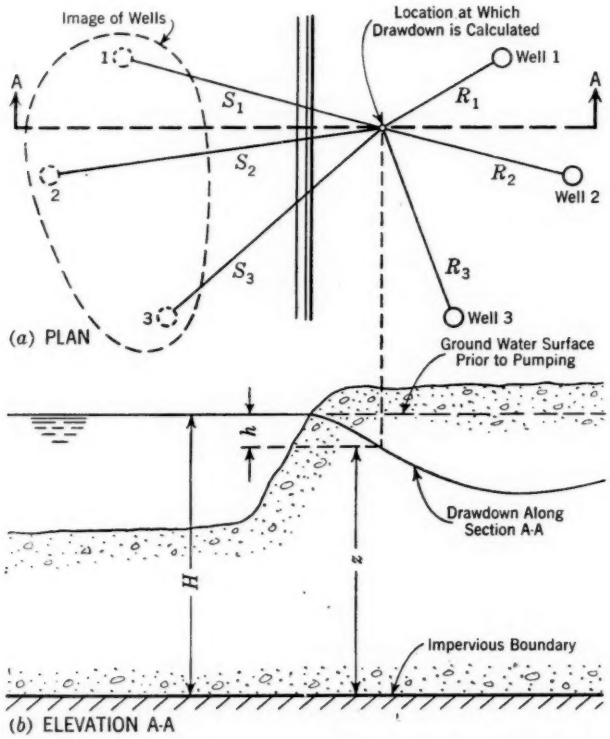

Fig. 1.—Series of Wells Adjacent to Open Water

from wells 1, 2, . . . n, respectively; $S_1, S_2, \ldots S_n$ is the distance from location at which drawdown is calculated to the image of wells 1, 2, . . . n, respectively; and $R_1, R_2, \ldots R_n$ is the distance from location at which drawdown is calculated to wells 1, 2, . . . n, respectively. This equation is based on six assumptions:

[2] "Hydraulik," by Phillip Forchheimer, B. G. Teubner, Leipzig and Berlin, 1930, pp. 82-90.

(*a*) The validity of J. Dupuit's assumptions that, for small inclinations of the free surface of a gravity flow system, the flow is horizontal and the velocity is proportional to the slope of the free surface;[3]

(*b*) The existence of a steady flow condition;

(*c*) The existence of a horizontal impervious boundary at depth H below the open water surface;

(*d*) A homogeneous and isotropic character for the pervious stratum;

(*e*) Horizontal ground-water surface prior to pumping; and

(*f*) Pervious well casings penetrating to the impervious boundary.

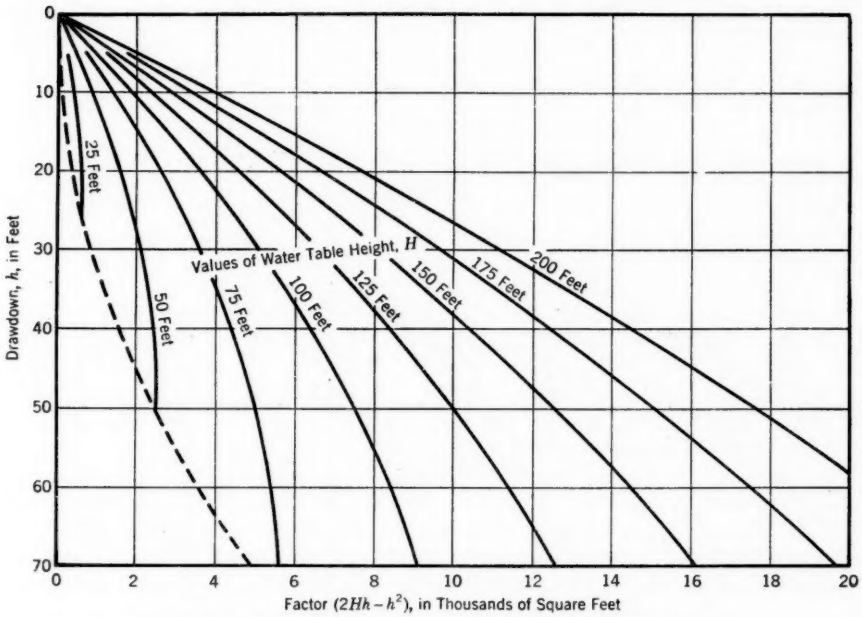

FIG. 2.—RELATION OF DRAWDOWN TO FACTOR ($2\,H\,h - h^2$)

Basic Relationship Between Drawdown and Rate of Pumping.—By making the following substitutions in Eq. 1: $q_T = q_1 + q_2 + \ldots q_n$; $c_1 = q_n/q_T$, $c_2 = q_2/q_T$, $c_n = q_n/q_T$; and $h = H - z$, in which h is the drawdown; the following equation for the total rate of pumping is obtained:

$$q_T = \frac{\pi\, k\, (2\,H\,h - h^2)}{\left(c_1 \log_e \frac{S_1}{R_1} + c_2 \log_e \frac{S_2}{R_2} + \cdots + c_n \log_e \frac{S_n}{R_n}\right)} \quad \ldots\ldots\ldots\ldots (2)$$

From this equation it is seen that for any arrangement of wells adjacent to open water, the total rate of pumping (q_T) is directly proportional to ($2\,H\,h - h^2$); and, when the drawdown ratio $\left(\frac{h}{H}\right)$ is small, the rate of pump-

[3] "Études théoriques et pratiques sur le mouvement des eaux," by J. Dupuit, 2d Ed., Paris, France, 1863.

ing (q_T) is almost directly proportional to the drawdown (h). This relationship is shown in Fig. 2, which facilitates estimating the value of q_T for various values of h when the value of q_T is known from observation for one value of h, provided that the ratios $c_1, c_2, \ldots c_n$ remain constant.

Drawdown at the Center of One Ring of Wells.—Fig. 3 shows a single ring of wells. This ring has a radius of r_1 with its center a distance p from the shore line. It is assumed that the total discharge is uniformly distributed along the ring among an infinite number of wells.

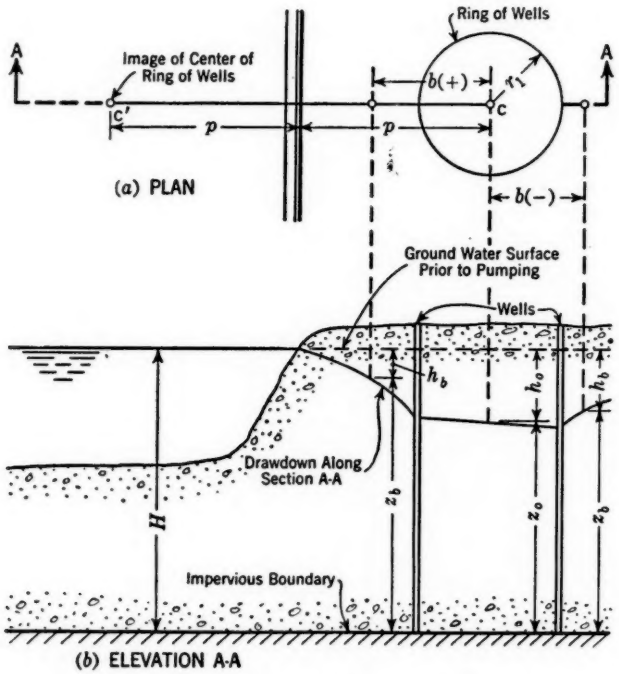

FIG. 3.—RING OF WELLS ADJACENT TO OPEN WATER

Assuming a differential distance along the ring, $r_1\, d\theta$, to represent one well (see Fig. 4), the differential equation for the drawdown at the center of the ring for this one well, from Eq. 1, is

$$z^2{}_o = H^2 - \frac{1}{\pi k}\left(\frac{q_I}{2\pi}\right)\left(\log_e \frac{\sqrt{4p^2 + r^2{}_1 - 4p r_1 \cos\theta}}{r_1}\right) d\theta \ldots\ldots (3)$$

in which z_o is the height of water surface at the center of the ring above impervious boundary, q_I is the total rate of pumping from all wells in ring I, assuming no other wells are being pumped, and θ is the polar angle at the center of ring I, measured clockwise from a normal to the shore line to any point on the ring.

To obtain the effect produced by an infinite number of wells in a ring, the differential term in Eq. 3 is integrated from 0 to 2π. This integration results in the equation of the drawdown surface at the center:

$$z^2_o = H^2 - \frac{1}{\pi k}\left(\frac{q_I}{2\pi}\right)\int_o^{2\pi} \log_e \frac{\sqrt{4p^2 + r^2_1 - 4p r_1 \cos\theta}}{r_1}\, d\theta \quad \ldots\ldots (4)$$

or,

$$z^2_o = H^2 - \frac{q_I}{\pi k}\log_e \frac{2p}{r_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

or, by introducing the drawdown at the center ($h_o = H - z_o$):

$$q_I = \frac{\pi k\,(2 H h_o - h^2_o)}{\log_e \dfrac{2p}{r_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

It follows from Eq. 6 that the total rate of pumping (q_I) from a ring of wells, I, is inversely proportional to $\log_e \frac{2p}{r_1}$. Fig. 5 is a series of curves

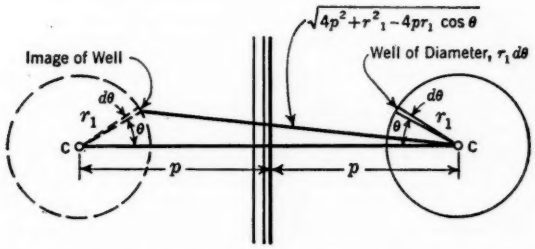

FIG. 4.—ASSUMPTION OF WELL OF DIFFERENTIAL DIAMETER IN A RING OF WELLS

computed from this relationship to facilitate computation of the rate of pumping necessary to produce a given drawdown at the center of any size ring of wells. In Fig. 5, q_I represents the total rate of pumping when ring I only is used and q_{II} is the total pumping rate when ring II alone is used. To use the curves it is necessary to know the required rate of pumping that will produce the same drawdown from one size ring of wells as that for the size of ring under study.

When it is necessary to vary both the drawdown and the radius, the required rate of pumping may be determined by using both Fig. 2 and Fig. 5.

Drawdown at the Center of Two or More Concentric Rings of Wells.—Fig. 6 shows two concentric rings of wells (I and II) with radii r_1 and r_2, having a common center at a distance p from the shore line. Similar to the derivation of Eq. 4, the equation for the drawdown at the center of the two rings is obtained:

$$z^2_o = H^2 - \frac{\bar{q}_I}{\pi k}\log_e \frac{2p}{r_1} - \frac{\bar{q}_{II}}{\pi k}\log_e \frac{2p}{r_2} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

in which $\bar{q}_I$ and $\bar{q}_{II}$ are the total rates of pumping from rings I and II, respect-

ively. By substituting $q_T = \bar{q}_I + \bar{q}_{II}$, $C_I = \bar{q}_I/q_T$, and $H - h_o = z_o$—

$$q_T = \frac{\pi\, k\, (2\, H\, h_o - h^2{}_o)}{\log_e \frac{2\,p}{r_2} - C_I \log_e \frac{r_1}{r_2}} \qquad (8)$$

This equation shows that for two rings of wells, the total rate of pumping, q_T, is inversely proportional to $\left(\log_e \frac{2\,p}{r_2} - C_I \log_e \frac{r_1}{r_2}\right)$. From this relationship it is possible to determine, for any desired value of the ratio C_I, the total rate of pumping (q_T) required from two concentric rings of wells to produce a given

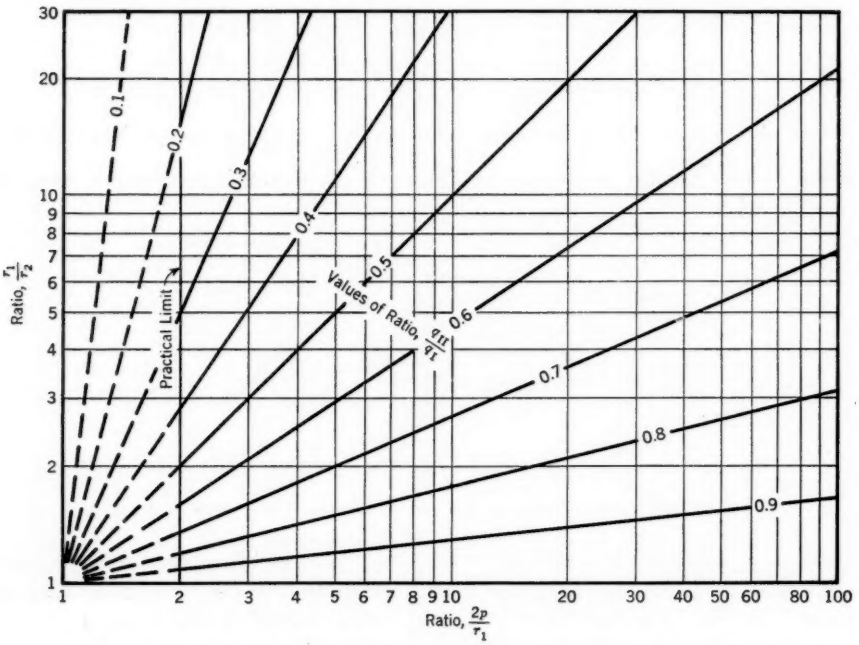

FIG. 5.—ESTIMATION OF PUMPING RATE FOR WELLS IN RINGS OF VARIOUS RADIUS

drawdown, provided the rate of pumping is known for the same drawdown when the outer ring is being pumped alone. First, if q_I and q_{II} are the required rates of pumping when the outer and inner rings, respectively, are being pumped alone, the ratio q_{II}/q_I may be obtained from Fig. 5. The second step requires the use of Fig. 7, a graph containing a series of curves facilitating the estimation of the required total rate of pumping from two concentric rings operating together. From this figure, the ratio q_T/q_I may be obtained for various values of the ratios q_{II}/q_I and C_I.

The general equation for the total rate of pumping for n concentric rings of wells is:

$$q_T = \frac{\pi\, k\, (2\, \mathrm{H}\, h_o - h^2{}_o)}{C_I \log_e \frac{2\,p}{r_1} + C_{II} \log_e \frac{2\,p}{r_2} + \cdots + C_n \log_e \frac{2\,p}{r_n}} \qquad (9)$$

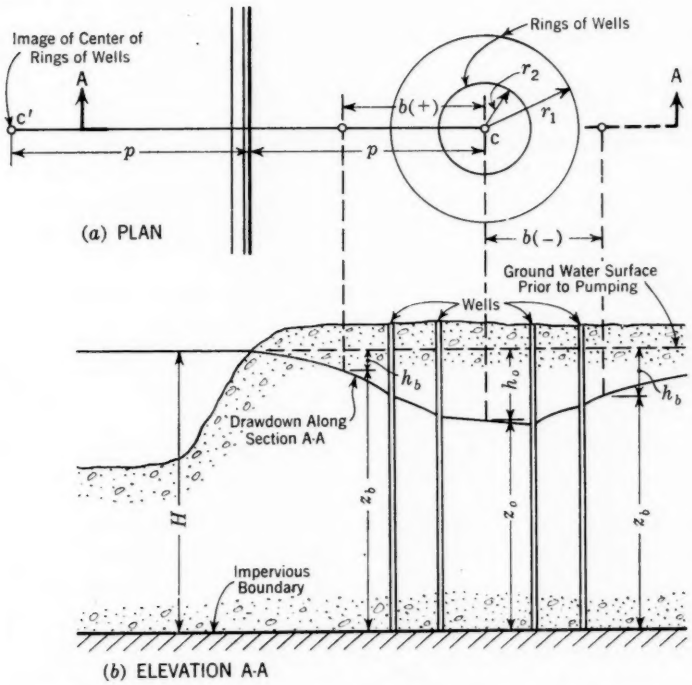

FIG. 6.—TWO CONCENTRIC RINGS OF WELLS ADJACENT TO OPEN WATER

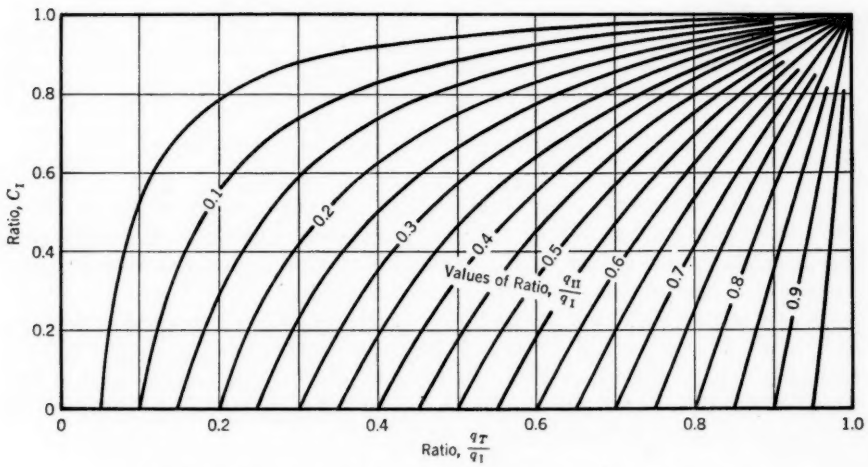

FIG. 7.—ESTIMATION OF PUMPING RATE FOR TWO CONCENTRIC RINGS OF WELLS OPERATING TOGETHER

in which $C_n = \bar{q}_n/q_T$. From this equation it is seen that the total rate of pumping is inversely proportional to $\left(C_{\text{I}} \log_e \frac{2p}{r_1} + C_{\text{II}} \log_e \frac{2p}{r_2} + \cdots + C_n \log_e \frac{2p}{r_n}\right)$. For any specific cases involving more than two rings it may be helpful to prepare graphs similar to Fig. 5 and Fig. 7.

Effect of a Rise or Fall in Water Stage on the Rate of Pumping.—It is often desired to maintain a constant drawdown, despite fluctuations in the level of adjacent open water. Where such fluctuations are of very short duration, their

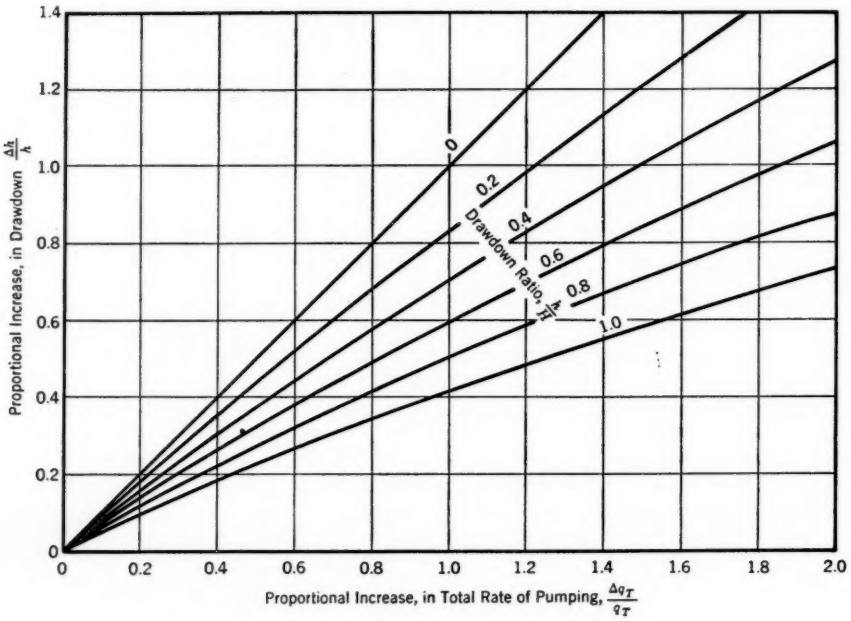

FIG. 8.—EFFECT OF RISE OF WATER STAGE ON PUMPING RATE

effect on the drawdown may be negligible. However, when a rise or fall in the open water level lasts long enough to establish a new steady condition of flow, a change is required in the total rate of pumping if the ground-water surface at a desired point is to remain unchanged.

If the rise or fall in open water level (ΔH) equals the change in drawdown (ΔH) and $H' = H + \Delta H$, $h' = h + \Delta h$, Δq_T is the change in total rate of pumping due to change in open water level, $q'_T = q_T + \Delta q_T$, $w = H/h$ and drawdown ratio $= h/H$, the change in the rate (Δq_T) may be expressed by the following equation, derived from Eq. 2:

$$\frac{\Delta q_T}{q_T} = \frac{\Delta h}{h}\left[1 + \frac{\frac{\Delta h}{h} + 1}{2w - 1}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Figs. 8 and 9 represent graphical solutions of Eq. 10 for rising and falling water stages, respectively. When the drawdown ratio is small, the rate of change in the pumping rate is approximately proportional to the rate of change in the drawdown, whereas for higher drawdown ratios the rate of change in the pumping rate is larger than the rate of change in the drawdown.

Drawdown Curve for a Single Ring of Wells.—Following the procedure used in deriving Eq. 5, the equation for the drawdown surface along a line through the center of the ring and perpendicular to the shore line is obtained:

for $b = p$ to r_1—

$$z^2_b = H^2 - \frac{q_I}{\pi k}\left[\log_e\left(\frac{2p}{b} - 1\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

for $b = r_1$ to $-r_1$—

$$z^2_b = H^2 - \frac{q_I}{\pi k}\left[\log_e \frac{2p - b}{r_1}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

for $b = -r_1$ to $-\infty$—

$$z^2_b = H^2 - \frac{q_I}{\pi k}\left[\log_e\left(1 - \frac{2p}{b}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (11c)$$

in which b is the distance from the center of the ring to a location at which the drawdown is calculated. It is considered positive when the location is on the shore line side of the center, and negative when on the other side, as shown in Fig. 3.

Substituting in Eq. 11: $H - h_b = z_b$ and $w = H/h_o$ the following equation for the relationship between the drawdown (h_b) and the drawdown at the center (h_o) is derived:

$$\frac{h_b}{h_o} = w - \sqrt{w^2 - (2w - 1)\frac{Z}{\log_e 2p}} \quad \ldots (12)$$

in which:

for $b = p$ to r_1—

$$Z = \log_e\left(\frac{2p}{b} - 1\right) \quad \ldots (13a)$$

for $b = r_1$ to $-r_1$—

$$Z = \log_e \frac{2p - b}{r_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

for $b = -r_1$ to $-\infty$—

$$Z = \log_e 1 - \frac{2p}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13c)$$

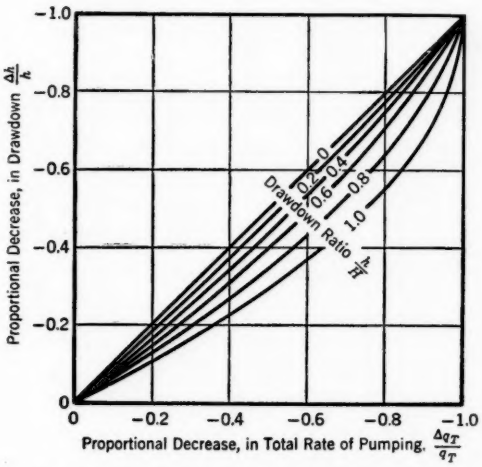

FIG. 9.—EFFECT OF FALL OF WATER STAGE ON PUMPING RATE

For the case in which the ring consists of a series of wells so widely spaced that they can no longer be considered to form a continuous ring of wells, Eq. 13 may be evaluated by letting:

$$Z = \sum_{1}^{n} \frac{1}{n} \log_e \frac{S_n}{R_n} \quad \ldots\ldots (14)$$

For an explanation of symbols, see Fig. 1.

Drawdown Curve for Two Concentric Rings of Wells.—In a manner similar to that used in the derivation of Eq. 11, the equation is derived for the drawdown surface along a line through the center of two concentric rings and perpendicular to the shore line:

for $b = p$ to r_1—

$$z^2_b = H^2 - \frac{q_T}{\pi k} \log_e \left(\frac{2p}{b} - 1 \right) \quad \ldots\ldots (15a)$$

for $b = r_1$ to r_2—

$$z^2_b = H^2 - \frac{\bar{q}_{\mathrm{I}}}{\pi k} \log_e \frac{2p - b}{r_1} - \frac{\bar{q}_{\mathrm{II}}}{\pi k} \log_e \left(\frac{2p}{b} - 1 \right) \quad \ldots\ldots (15b)$$

for $b = r_2$ to $-r_2$—

$$z^2_b = H^2 - \frac{\bar{q}_{\mathrm{I}}}{\pi k} \log_e \frac{2p - b}{r_1} - \frac{\bar{q}_{\mathrm{II}}}{\pi k} \log_e \frac{2p - b}{r_2} \quad \ldots\ldots (15c)$$

for $b = -r_2$ to $-r_1$—

$$z^2_b = H^2 - \frac{\bar{q}_{\mathrm{I}}}{\pi k} \log_e \frac{2p - b}{r_1} - \frac{\bar{q}_{\mathrm{II}}}{\pi k} \log_e \left(1 - \frac{2p}{b} \right) \quad \ldots\ldots (15d)$$

for $b = -r_1$ to $-\infty$—

$$z^2_b = H^2 - \frac{q_T}{\pi k} \log_e \left(1 - \frac{2p}{b} \right) \quad \ldots\ldots (15e)$$

From this equation it follows:

$$\frac{h_b}{h_o} = w - \sqrt{w^2 - (2w - 1) \frac{Z}{\log_e \frac{2p}{r_2} - C_{\mathrm{I}} \log_e \frac{r_1}{r_2}}} \quad \ldots\ldots (16)$$

in which

for $b = p$ to r_1—

$$Z = \log_e \left(\frac{2p}{b} - 1 \right) \quad \ldots\ldots (17a)$$

for $b = r_1$ to r_2—

$$Z = \log_e \left(\frac{2p}{b} - 1 \right) - C_{\mathrm{I}} \log_e \frac{r_1}{b} \quad \ldots\ldots (17b)$$

for $b = r_2$ to $-r_2$—

$$Z = \log_e \frac{2p - b}{r_2} - C_{\mathrm{I}} \log_e \frac{r_1}{r_2} \quad \ldots\ldots (17c)$$

for $b = -r_2$ to $-r_1$—

$$Z = \log_e\left(1 - \frac{2p}{b}\right) - C_I \log_e \frac{-r_1}{b} \quad \ldots\ldots\ldots\ldots (17d)$$

for $b = -r_1$ to $-\infty$—

$$Z = \log_e\left(1 - \frac{2p}{b}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (17e)$$

In order to determine the drawdown for any point at a distance y from the center of the rings and a distance x from the image point of the center, the same general equation (Eq. 16) applies except for the values of Z that are:

for $y = 0$ to r_2—

$$Z = \log_e \frac{x}{r_2} - C_I \log_e \frac{r_1}{r_2}$$

for $y = r_2$ to r_1—

$$Z = \log_e \frac{x}{y} - C_I \log_e \frac{r_1}{y} \qquad (17f)$$

for $y = r_1$ to ∞—

$$Z = \log_e \frac{x}{y}$$

Within the inner ring the drawdown is independent of the distance from the center of the rings (y). Consequently, contour lines of the ground-water surface within this ring are arcs of circles with a common center at the image point of the center of the well system.

Nonconcentric Rings of Wells with Their Centers on a Line Perpendicular to the Shore Line.—It is sometimes necessary to dewater two separate areas that have their centers on a line perpendicular to the shore line, as shown in Fig. 10. Replacing the actual shape of the well installations by circles, the ratio between the drawdowns at the centers of the two rings may be expressed

$$\frac{h_{bo}}{h_{ao}} = w_a - \sqrt{w^2_a - (2\,w_a - 1)\left[\frac{\log_e\left(\frac{2p}{f} - 1\right) + C_{b/a}\log_e \frac{2\,(p-f)}{r_b}}{\log_e \frac{2p}{r_a} + C_{b/a}\log_e\left(\frac{2p}{f} - 1\right)}\right]} \quad \ldots (18)$$

in which ring b is between ring a and the shore line, $w_a = H/h_{ao}$, $C_{b/a} = \bar{q}_b/\bar{q}_a$, $\bar{q}_a$ is the rate of pumping from ring a, $\bar{q}_b$ is the rate of pumping from ring b, p is the distance from the center of ring a to the shore line, and f is the distance between the centers of rings a and b; or, solving for $C_{b/a}$:

$$C_{b/a} = \frac{(1 - 2\,w_a)\log_e\left(\frac{2p}{f} - 1\right) - \frac{h_{bo}}{h_{ao}}\left(\frac{h_{bo}}{h_{ao}} - 2\,w_a\right)\log_e \frac{2p}{r_a}}{\frac{h_{bo}}{h_{ao}}\left(\frac{h_{bo}}{h_{ao}} - 2\,w_a\right)\log_e\left(\frac{2p}{f} - 1\right) - (1 - 2\,w_a)\log_e \frac{2\,(p-f)}{r_b}} \quad \ldots (19)$$

The ratio of the drawdown at any point outside the two rings to the drawdown

at the center of ring a is:

$$\frac{h}{h_{ao}} = w_a - \sqrt{w^2_a - (2\,w_a - 1)\left[\frac{\log_e \frac{x_a}{y_a} + C_{b/a} \log_e \frac{x_b}{y_b}}{\log_e \frac{2\,p}{r_a} + C_{b/a} \log_e \left(\frac{2\,p}{f} - 1\right)}\right]} \quad \ldots (20)$$

in which: x_a and x_b are the distance from the location at which the drawdown is being calculated to the image point of the center of rings a and b, respectively, and y_a and y_b are the distance to the center of rings a and b, respectively.

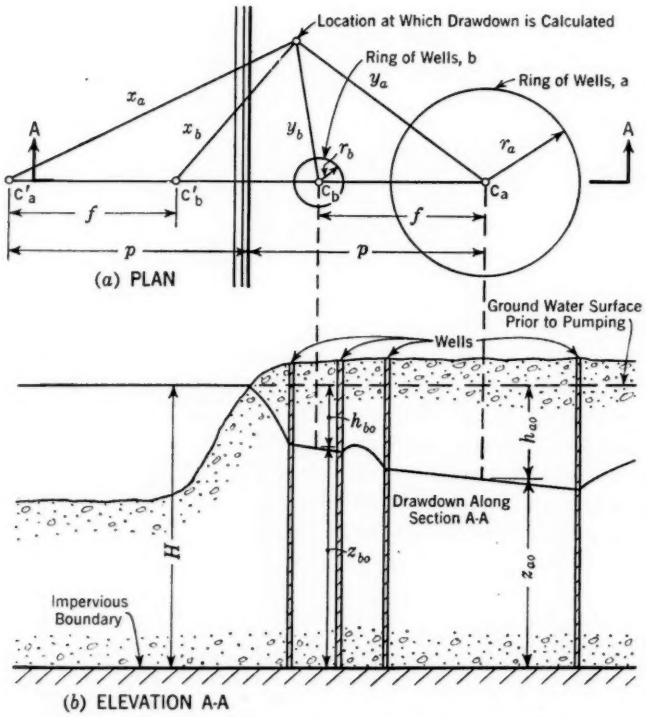

FIG. 10.—TWO NONCONCENTRIC RINGS OF WELLS ADJACENT TO OPEN WATER

In the course of his studies, the writer found it necessary to solve the following problem: Ring a has been in operation and the required drawdown (h_{ao}) and the rate (q_a) have been obtained, when pumping from ring a alone. It is desired to install a ring b in order to obtain a drawdown h_{bo} at its center. The required rate of pumping ($\bar{q}_b$) for this ring can be determined by the following procedure:

1. Solve Eq. 19 for $C_{b/a}$.
2. Solve the following equation for $\bar{q}_a$, the rate required for ring a when

ring b is also in operation:

$$\bar{q}_a = \frac{q_a}{1 + C_{b/a} \dfrac{\log_e \left(\dfrac{2p}{f} - 1 \right)}{\log_e \dfrac{2p}{r_a}}} \qquad (21)$$

3. $\bar{q}_b = C_{b/a}\, \bar{q}_a$.

The rate of pumping for other arrangements of nonconcentric rings of wells may be determined also by deriving equations similar to those given.

Examples of Drawdown Curves

Typical computed drawdown curves along a line perpendicular to the shore line and through the center of concentric rings of wells adjacent to open

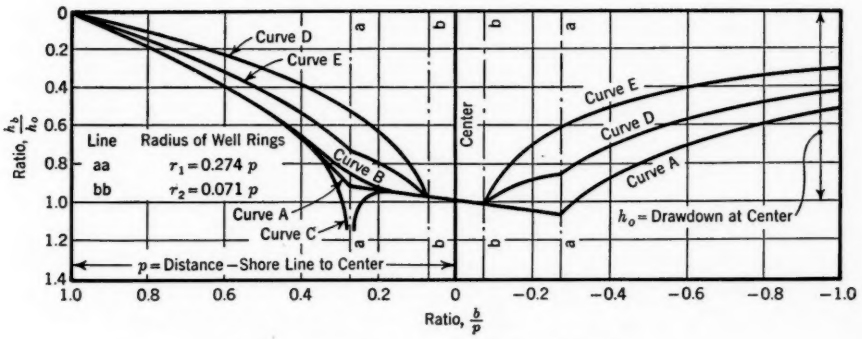

Fig. 11.—Typical Computed Drawdown Curves

water are shown in Fig. 11 for $H/h_o = 4.5$. The curves in Fig. 11 are identified as follows:

Curve	Description
A	Drawdown when pumping only from an infinite number of wells in outer ring of radius r_1
B	Drawdown midway between adjacent wells when pumping from six equally spaced wells in ring of radius r_1. (See Fig. 12 (*a*).)
C	Drawdown directly through two wells when pumping from six equally spaced wells in a ring of radius r_1 (see Fig. 12 (*b*).)
D	Drawdown when pumping $\frac{1}{3}$ from an infinite number of wells in inner ring of radius r_2 and $\frac{2}{3}$ from an infinite number of wells in outer ring of radius r_1
E	Drawdown when pumping only from an infinite number of wells in inner ring of radius r_2

The computed drawdown within the inner ring in all cases is practically a plane surface, with a slight dip away from the shore line.

When a single ring consists of a small number of wells only, the drawdown curve in the vicinity of the ring varies from the plane surface. Note in Fig. 11 that midway between adjacent wells the drawdown, Curve B, is higher than the drawdown, Curve A, for a continuous ring of wells; whereas in the vicinity of the well the drawdown, Curve C, is lower. As the number of wells in the ring is increased, the drawdown at the well and midway between adjacent wells (Curves C and B respectively), approach the drawdown curve for an infinite number of wells in the ring (Curve A).

Assuming the same drawdown at the center, the relative rates of pumping required to produce the drawdown curves, A, D, and E, are as follows:

Curve	Description	Relative rate of pumping
A	Pumping only from ring I with radius 0.274 p	1.00 q_I
D	Pumping $\frac{2}{3}$ from ring I and $\frac{1}{3}$ from ring II	0.81 q_I
E	Pumping only from ring II with radius 0.071 p	0.60 q_I

The foregoing rates are for a particular case. In general the ratio between these rates of pumping is dependent only on the following: (1) Ratio of the radius of ring I to the distance from the center of the ring to the shore line; (2) Ratio of the radius of ring II to the distance from the center of the ring

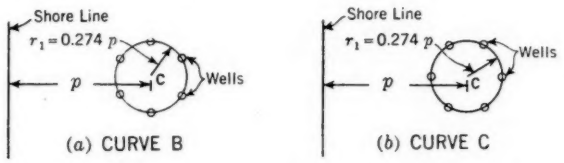

FIG. 12.—WELL LAYOUT FOR DRAWDOWN CURVES B AND C IN FIGURE 11

to the shore line; and (3) Ratio between the rates of pumping from each ring of wells, when they are being pumped together.

ANALYSIS OF AN ACTUAL INSTALLATION OF WELLPOINTS ADJACENT TO A RIVER

The writer made an analysis of a wellpoint installation, consisting of three stages of wellpoints, as shown in Fig. 13(*a*), that were used to dewater a power house site on sand and gravel adjacent to a river. The first stage of wellpoints consisted of 148 points at an average spacing of 5 ft. In the second stage there were 215 points at an average spacing of 3 ft, and the third stage had 67 points at an average spacing of 3 ft. For purposes of analysis the roughly square arrangements of wellpoints were assumed circular as shown in Fig. 13(*b*), with a common center and each circle enclosing approximately the same area as the approximate square it replaces.

Since no pumping tests were made at this site prior to the installation of the first stage of wellpoints, no analysis was made until after the first stage had been placed in operation. This stage, for convenience, is called operation No. 1. Operation No. 2 consists of pumping only from the second stage of wellpoints and operation No. 3 is pumping $\frac{2}{3}$ of the total rate from the second stage and $\frac{1}{3}$ from the third stage. Observations made on each pumping

operation were used to compute in advance the rate of pumping required for the succeeding operation. The computed rates varied from the actual measured rates by 4% for operation No. 2 and 14% for operation No. 3. The effective coefficient of permeability computed from the three operations is given in Table 1.

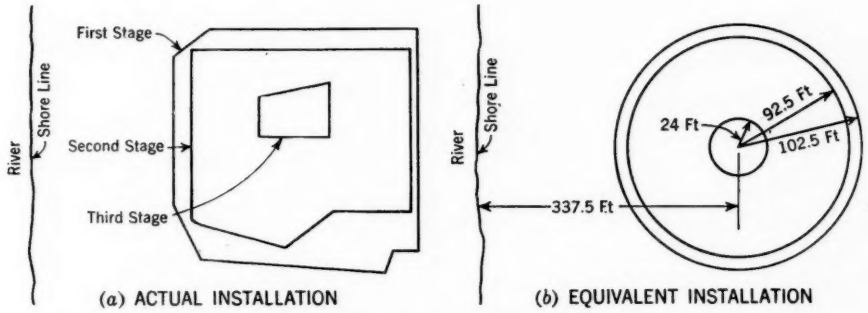

FIG. 13.—WELLPOINT INSTALLATION ADJACENT TO RIVER

The drawdown curves normal to the river for the three pumping operations are shown in Fig. 14. The drawdowns at the center of the installation and the river stages were plotted from actual observations, whereas the remainder of the drawdown curves were computed. The observed drawdown in the vicinity of the wellpoints varies slightly from the computed value as shown in the figure. These observations indicate that, for operations No. 1 and No. 2, the drawdown inside the ring of wellpoints was roughly a plane surface dipping slightly toward the river rather than away from the river as indicated in the analysis. The difference is probably due to the fact that the pumps for all stages were connected to the headers on the river side of the installation and consequently, because of larger friction losses in the headers, the yield from the wellpoints on the land side was not so large as that from the wellpoints on the river side. Corrections could be made in the computations of drawdown for uniform variations of this type in the rate of pumping, if the variations were considered large enough to justify the additional computations.

TABLE 1.—EXAMPLE OF WELLPOINT INSTALLATION

Operation number	Effective coefficient of permeability (ft per min)	RATES OF PUMPING (GAL PER MIN)		
		Measured	Computed	Error (%)
1	0.092	2578	....	....
2	0.096	4407	4210	− 4
3	0.083	4546	5180	+14

The drawdown during operation No. 3, at a point approximately 500 ft from the wellpoint installation, was computed to be within a fraction of a foot of the actual observed drawdown at this point.

In anticipation of flood stages in the river, curves for this site showing the rates of pumping required to maintain various drawdowns for various river stages were prepared. These curves, computed on the assumption of a steady flow condition, indicated that the rate of increase in the pumping rate would be

about proportional to the rate of increase in the drawdown caused by the rise in the river stage. Observations made during flood stages prior to pumping indicated that it takes about a week for the full effect of a rise in the river stage to be reflected in the ground water at this site. Consequently, the river stage would have to remain constant for this same length of time before a condition of steady flow is reached.

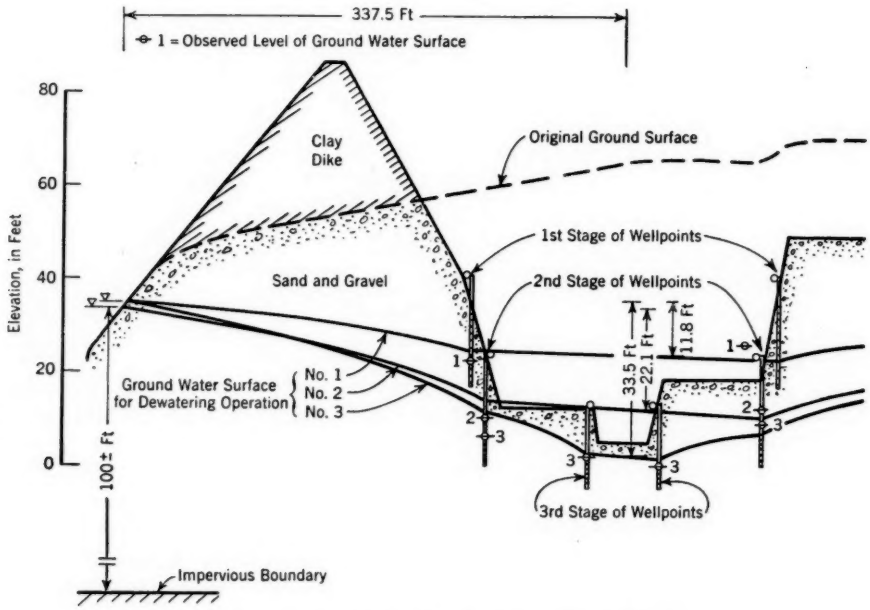

FIG. 14.—SECTION THROUGH CENTER OF WELLPOINT INSTALLATION

Several high-water river stages occurred at the site during the course of construction, including floods of about 40 ft above normal. In all cases, from four days to a week were required before a condition of steady flow was reached after a sudden large change in river stage. However, whenever a steady flow condition existed the measured pumping rates were almost identical with the computed values.

CONCLUSION

Many simplifications are required in the assumptions upon which this analysis is based, in order that simple, workable formulas can be derived. Consequently, errors up to 10% or 15% are to be expected in estimates of rates of pumping and drawdowns. The writer, however, believes that computed estimates within these limits are desirable on all large well installations.

ACKNOWLEDGMENT

The writer is indebted to Mr. José Corso, Jun. M. ASCE, who checked the derivation of all the formulas, and to A. Casagrande, M. ASCE, for numerous suggestions and encouragement.

CURRENT PAPERS AND DISCUSSIONS

Proceedings-Separate Number	Date Available	Title and Author	Discussion closes
72	July, 1951	"Design of Large Coal Bunkers," by Paul Rogers	Dec. 1
73	July, 1951	"Health Implications of Air Pollution," by J. J. Bloomfield	Dec. 1
74	July, 1951	"Acceptable Standards for Natural Waters Used for Bathing," by Charles R. Cox	Dec. 1
75	July, 1951	"Base Course Drainage for Airport Pavements," by Arthur Casagrande and William L. Shannon	Dec. 1
76	July, 1951	"Model Tests Using Low-Velocity Air," by James W. Ball	Dec. 1
77	Aug., 1951	"Buckling Stresses for Flat Plates and Sections," by Elbridge Z. Stowell, George J. Heimerl, Charles Libove, and Eugene E. Lundquist	Jan. 1
78	Aug., 1951	"River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa	Jan. 1
79	Aug., 1951	"Stage Predictions for Flood Control Operations," by Ralph E. King	Jan. 1
80	Aug., 1951	"Mississippi River Valley Geology Relation to River Regime," by Harold N. Fisk	Jan. 1
81	Aug., 1951	"Petenwell Hydroelectric Project," by E. Montford Fucik	Jan. 1
82	Sept., 1951	"Pressures in a Shallow Rectangular Bin," by Raymond L. Moore and J. R. Shaw	Feb. 1
83	Sept., 1951	"Waterway Traffic on the Great Lakes," by John R. Hardin	Feb. 1
84	Sept., 1951	"Longitudinal Mixing Measured by Radioactive Tracers," by Harold A. Thomas, Jr., and Ralph S. Archibald	Feb. 1
85	Sept., 1951	"Resinous Ion Exchangers in Water Treatment," by William W. Aultman	Feb. 1
86	Sept., 1951	"Ground-Water Movement Controlled Through Spreading," by Paul Baumann	Feb. 1
87	Oct., 1951	"Sewage Reclamation by Spreading Basin Infiltration," by Ralph Stone and William F. Garber	Mar. 1
88	Oct., 1951	"Experimental Study of Water Flow in Annular Pipes," by W. M. Owen	Mar. 1
89	Oct., 1951	"Deflections in Gridworks and Slabs," by Walter W. Ewell, Shigeo Okubo, and Joel I. Abrams	Mar. 1
90	Nov., 1951	"Consumptive Use of Water by Forest and Range Vegetation," by L. R. Rich	Apr. 1
91	Nov., 1951	"Consumptive Use of Water," by Harry F. Blaney	Apr. 1
92	Nov., 1951	"Experimental Investigation of Fire Monitors and Nozzles," by Hunter Rouse, J. W. Howe, and D. E. Metzler	Apr. 1
93	Nov., 1951	"Aircraft Design as Related to Airport Standards," by Milton W. Arnold	Apr. 1
94	Nov., 1951	"Friendship International Airport," by Benjamin Everett Beavin	Apr. 1
95	Nov., 1951	"Directional Requirements for Airport Runways," by Ralph H. Burke and Harry Otis Wright, Jr.	Apr. 1
96	Nov., 1951	"Surface Curves for Steady Nonuniform Flow," by Robert B. Jansen	Apr. 1
97	Dec., 1951	"Consumptive Use in the Rio Grande Basin," by Robert L. Lowry	May 1
98	Dec., 1951	"Consumptive Use of Water on Irrigated Land," by Wayne D. Criddle	May 1
99	Dec., 1951	"Consumptive Use in Municipal and Industrial Areas," by George B. Gleason	May 1
100	Dec., 1951	"Forced Vibrations of Continuous Beams," by Edward Saibel and Elio D'Appolonia	May 1
101	Dec., 1951	"Application of Highway Capacity Research," by J. P. Buckley	May 1
102	Dec., 1951	"Utilization of Ground Water in California," by T. Russel Simpson	May 1
103	Dec., 1951	"Pile Foundations for Large Towers on Permafrost," by L. A. Nees	May 1
104	Dec., 1951	"Redesign of Major Airport Terminals," by Herbert H. Howell	May 1
105	Dec., 1951	"Principles of Highway Capacity Research," by O. K. Normann	May 1
106	Jan., 1952	"Analysis of Ground-Water Lowering Adjacent to Open Water," by Stuart B. Avery, Jr.	June 1
107	Jan., 1952	"Characteristics of the Solitary Wave," by James W. Daily and Samuel C. Stephan, Jr.	June 1
108	Jan., 1952	"Control of Embankment Material by Laboratory Testing," by F. C. Walker and W. G. Holtz	June 1
109	Jan., 1952	"Final Foundation Treatment at Hoover Dam," by A. Warren Simonds	June 1
110	Jan., 1952	"Review of Flood Frequency Methods," Final Report of the Subcommittee of the ASCE Joint Division Committee on Floods	June 1
111	Jan., 1952	"Research in Water Spreading," by Dean C. Muckel	June 1

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1951

PRESIDENT

GAIL A. HATHAWAY

VICE-PRESIDENTS

Term expires January, 1952:

FRED C. SCOBEY
ALBERT HAERTLEIN

Term expires October, 1952:

WILLIAM R. GLIDDEN
DANIEL V. TERRELL

DIRECTORS

Term expires January, 1952:

WALDO G. BOWMAN
HAROLD L. BLAKESLEE
PAUL L. HOLLAND
EDMUND FRIEDMAN
S. T. HARDING

Term expires January, 1953:

OTTO HOLDEN
FRANK L. WEAVER
GORDON H. BUTLER
G. BROOKS EARNEST
WALTER J. RYAN*
GEORGE W. LAMB

Term expires October, 1952:

MORRIS GOODKIND
MILTON T. WILSON

Term expires October, 1953:

KIRBY SMITH
FRANCIS S. FRIEL
LOUIS R. HOWSON
WALLACE L. CHADWICK
NORMAN R. MOORE
BURTON G. DWYRE

PAST-PRESIDENTS

Members of the Board

FRANKLIN THOMAS
ERNEST E. HOWARD

TREASURER
CHARLES E. TROUT

EXECUTIVE SECRETARY
WILLIAM N. CAREY

ASSISTANT TREASURER
GEORGE W. BURPEE

ASSISTANT SECRETARY
E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Technical Publications

HAROLD T. LARSEN
Editor of Technical Publications

JOHN C. REHFIELD
Assoc. Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

WALDO G. BOWMAN

FRANCIS S. FRIEL
S. T. HARDING
OTTO HOLDEN
LOUIS R. HOWSON
NORMAN R. MOORE

* Deceased.